Norges Bank Investment Management

#2 | 2023

Discussion note

Withdrawals from the GPFG and potential trade-offs

We outline a simulation model of the Norwegian economy and the Government Pension Fund Global. We explore the trade-off between spending from the fund in line with expected returns, and spending counter-cyclically in Norway. We analyse the cyclical properties and sustainability of a range of alternative rules for guiding withdrawals from the fund.

Date 23/08/2023

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Summary

- The Norwegian fiscal policy framework describes a spending rule that guides withdrawals from the Government Pension Fund Global (GPFG). Withdrawals are guided by the expected return on the fund, while also accommodating counter-cyclical fiscal deficits in Norway. In recent years, these withdrawals have financed growing fiscal deficits and a significant proportion of total fiscal expenditure.
- In this note, we extend an asset price simulation model presented in Discussion Note #1 | 2023 to include the Norwegian economy. The extended model includes equity and fixed income returns, the Norwegian fiscal budget, the Norwegian Krone exchange rate, and petroleum revenues. We simulate the evolution of GPFG and the Norwegian fiscal budget given today's level of expected returns and fiscal deficits. We generate distributions of the real value of the fund and paths for fund withdrawals, based on different spending rules.
- We use the model to understand trade-offs when spending from the fund, over short and long horizons. We highlight a long-term trade-off between spending *sustainably* and spending *cyclically*. Sustainable fund withdrawals need to be set in line with expected returns, but this can imply deficit reductions during economic downturns in Norway. On the other hand, when spending counter-cyclically in response to downturns, the probability of depleting the real value of the fund grows over time.
- We analyse commonly proposed rules for guiding fund withdrawals, such as spending cash flows the fund generates, or smoothing spending over time. These rules tend to stabilise spending and achieve a reasonable level of fund preservation, but imply spending patterns that do not align with cyclical fiscal spending needs.
- We outline a rule that attempts to strike a balance between spending cyclically and sustainably. This rule sets withdrawals directly in line with cyclical spending, but targets reductions in the fiscal deficit over time. These gradual reductions in deficits help to improve the distribution of fund over the longer term, and can offset the increased risk of depletion associated with counter-cyclical spending.

1. Introduction

The Government Pension Fund Global (GPFG) plays a key role in supporting fiscal expenditure in Norway. Since 2001, a fiscal 'spending rule' has been in place, which guides withdrawals of capital from the fund, and finances the Norwegian fiscal budget deficit. According to this rule, withdrawals from the fund should align with the expected real return on the fund, in order to preserve the real value of the fund over time. In addition, the rule states that withdrawals should be counter-cyclical, since fiscal deficits tend to increase during downturns.

In this note, we use a simulation framework to understand the trade-off between two aims of the fiscal spending rule. We refer to the first aim - to preserve the real value of the fund over the long-term - as spending *sustainably* from the fund. We refer to the second aim - to support counter-cyclical spending in Norway - as spending *cyclically* from the fund. To explore this trade-off, we simulate the Norwegian economy and GPFG portfolio. The model extends the simulation model of US fixed income and equity returns, presented in NBIM (2023), to include the Norwegian economy. We model the Norwegian fiscal budget alongside the Norwegian Krone exchange rate and petroleum revenues. We simulate the evolution of the fund, including inflows and withdrawals, over short and long horizons.

Over the past two decades, the value of the GPFG has increased substantially, and fiscal deficits/withdrawals from the fund have also increased. High financial returns and net inflows into the fund have meant that spending in line with the expected return on the fund has been accompanied by increasing withdrawals. Over the past few years, withdrawals have averaged near to 3% of the value fund. These withdrawals have equated to deficits near to 10% of mainland Norway GDP, equal to around 20% of total fiscal spending. Given this current context, it is important to understand the risks from continuing to use fund withdrawals to finance significant proportions of fiscal spending. Declines in expected returns, or the value of the fund, could mean fiscal spending needs to adjust downward to ensure withdrawals remain sustainable. Another risk is that repeated periods of large counter-cyclical spending from the fund, such as during and after the COVID-19 pandemic, can increase the likelihood of depleting the fund over the long term.

Using the simulation model, we show how fund withdrawals differ when focusing on either sustainable or cyclical spending in isolation. Sustainable spending implies setting fund withdrawals in line with expected real returns, while cyclical spending sets withdrawals in line with the Norwegian business cycle. We show that there is a significant probability that these two approaches do not coincide. Spending sustainably often implies reducing withdrawals below levels required to support the Norwegian economy. Alternatively, when spending in line with the Norwegian economic cycle, there is a high probability of depleting the real value of the fund. While this risk is negligible over horizons of a few years, the risk of depletion increases substantially over the long term.

In light of this trade-off, we assess two commonly proposed approaches to guiding withdrawals from the fund: spending cash flows from the portfolio, and smoothing spending over time. These approaches cannot simultaneously provide sustainable and counter-cyclical withdrawals. They tend to improve the stability of fund withdrawals, and perform well in terms of preserving the value of the fund. However, neither approach accommodates cyclical spending. In particular, the spending of cash flows implies reductions in fiscal deficits to lower levels, and spending in a way that is not closely aligned with the Norwegian economy.

We outline an alternative approach that aims to accommodate counter-cyclical withdrawals, while also preserving long-term values of the fund. Withdrawals are set directly in line with deficits implied by the Norwegian economic cycle, and they do not directly depend on the value of the fund. To reduce the risk of fund depletion associated with spending counter-cyclically, the rule targets gradual reductions in the fiscal deficit over time. These reductions improve the distribution of the fund over the longer term, and the size and pace of reductions can be adjusted to balance risks across the short and long term.

In the next section, we provide background on the fiscal policy framework in Norway and the role of the fund. This highlights the range of moving parts involved in modelling the fund and Norway. In Section 3, we describe the simulation model and its calibration. The model necessarily contains many different processes, and its design is informed by several different areas of past research. Readers that are primarily interested in the mechanics of different spending approaches could skip directly to Section 4. In that section, we discuss the trade-off between sustainable and cyclical withdrawals, and show this trade-off using the simulation model. Section 5 uses the model to understand alternative rules, and how they perform in terms of preserving the fund versus meeting cyclical spending needs. Section 6 concludes.

2. The fiscal policy framework over the past two decades

The fiscal policy framework describes how two sources of income, investment returns and petroleum revenues, are incorporated into the Norwegian economy. Petroleum revenues that the state receives are paid directly into GPFG, and since 2001, a 'spending rule' has guided the withdrawals from the fund.[1] These withdrawals are transferred to the central government budget, where they are used to finance the fiscal budget deficit in its entirety.[2]

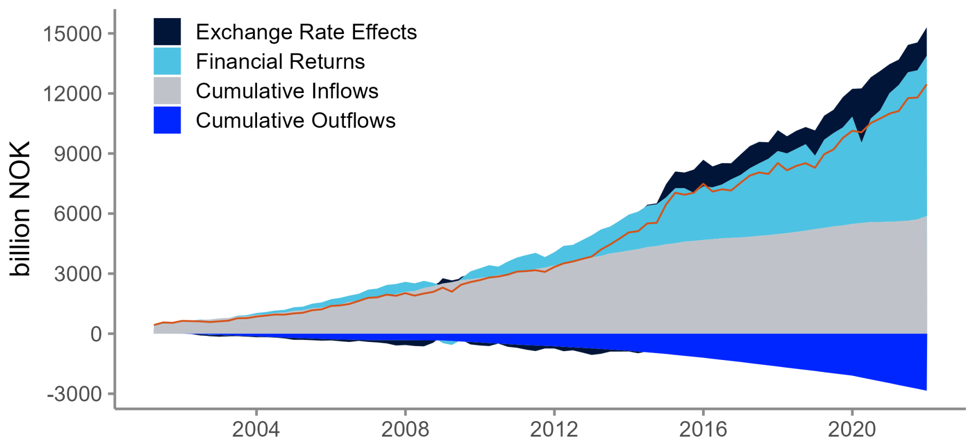
The spending rule states that withdrawals from the fund to the budget should, over time, follow the expected real return on the fund. This guidance aims to ensure that the real value of the fund is preserved over the long term. In addition, the rule emphasises that withdrawals should be used to even out fluctuations in the Norwegian economy, covering larger fiscal deficits during downturns.

Over the past two decades, the value of GPFG has increased substantially, and withdrawals from the fund have also increased. The growth in the value of the fund

[1] The rule is not a legal requirement, but rather a guiding principle for fund withdrawals.
[2] Specifically, the 'non-oil' budget deficit, which excludes petroleum revenues and expenditure from its calculation.

is attributable to several sources, shown in Figure 1. High financial returns on the fund's investment holdings account for the majority of accumulated wealth, which has further increased as a result of net inflows of petroleum revenues. In addition, the Norwegian Krone value of the fund is higher today due to a weaker exchange rate relative to the currencies of the fund's international investments.

FIGURE 1 Decomposition of the value of GPFG into cumulative inflows, outflows and return contributions

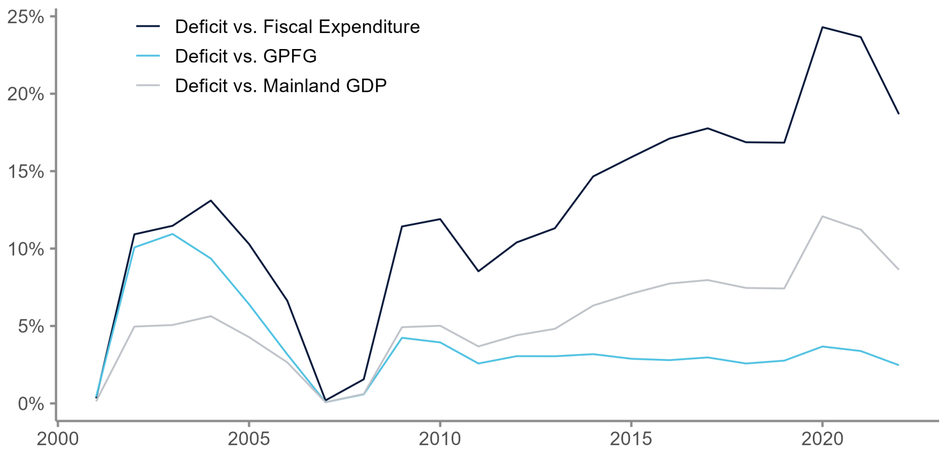


NOTE: The chart decomposes the value of the GPFG over the period from Q1 2001 to Q4 2021. The solid line depicts the value of the fund. The chart also includes cumulative contributions of petroleum revenue inflows, outflows to cover the non-oil fiscal deficit, investment returns and exchange rate effects. Annual GPFG management fees are included in cumulative outflows, but are a small component. Calculations are approximate and the decomposition is for illustrative purposes.

Figure 2 shows annual withdrawals from the fund since 2001, expressed as a percentage of the fund's value, mainland GDP in Norway, and total fiscal expenditure. While withdrawals have been relatively stable as a proportion of the fund in recent years, they have been increasing over time as the fund has grown. In addition, withdrawals have supported counter-cyclical spending, such as during the 2008-09 financial crisis and the COVID-19 pandemic. The fund has grown more quickly than mainland GDP and total fiscal expenditure, meaning that the fiscal deficit has also grown as a proportion of GDP. Over the past few years, withdrawals have averaged around 3% of the fund, covering a deficit near to 10% of GDP, and contributing over 20% to total fiscal spending.

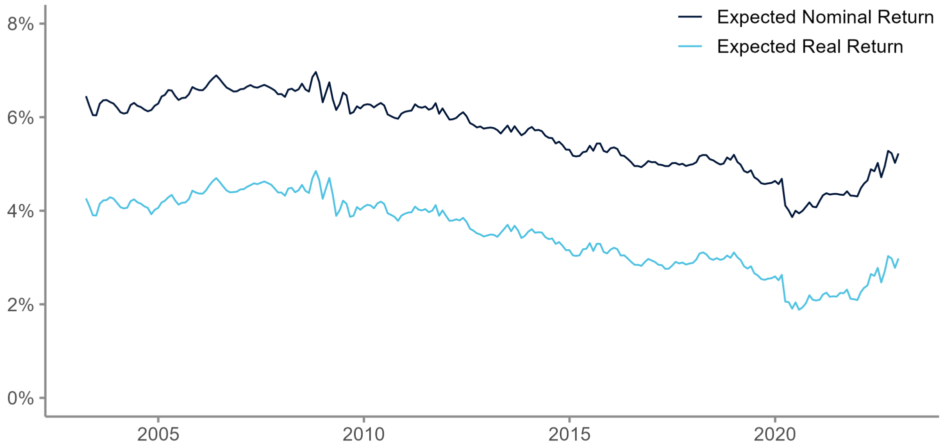
Increasing fund withdrawals have been possible despite declining expected returns on the fund. When introduced in 2001, the guiding expected real rate of return on the GPFG was set at 4%. This value was then reduced to 3% in 2017. Figure 3 shows the path for the long-term expected return, using the estimates from NBIM (2022). Despite declining expected returns, a downward adjustment to fiscal expenditure has not been required. In fact, excluding the early sample and the two crisis episodes, actual fund withdrawals have often been lower than the expected return guidance. This has been possible since declining expected returns, in combination with oil revenues, have generated large increases in the

FIGURE 2 Withdrawals from GPFG - the budget deficit - as a proportion of fund value and fiscal budget, and mainland GDP



NOTE: Chart shows non-oil fiscal deficit relative to central government expenditure, the value of GPFG at the beginning of each year, and mainland GDP in Norway for the period 2001-2022. Data are sourced from Statistics Norway and the National Budget 2023. Fiscal expenditure excludes petroleum activities.

FIGURE 3 Expected returns on the fund



NOTE: Figure shows long-horizon expected returns on the fund's portfolio, based on the methodology in NBIM (2022). Sample period is January 2003 to December 2022.

value of the fund.[3] Over the past two decades, withdrawals appear to have been set sustainably, while also meeting counter-cyclical spending needs during downturns. A key question is whether we should expect the same experience going forward. The historical experience so far has been relatively benign, where appreciation of the fund's investments and oil revenues have provided room for increasing fiscal deficits.

[3] NBIM (2022) and NBIM (2023) discuss the relationship between realised and expected returns. In particular, they highlight persistent shifts in expected returns that are negatively related to realised returns on long-duration assets such as equities and government bonds. They also argue that investment outcomes of the past two decades are unlikely to repeat.

In order to evaluate these risks going forward, we use a simulation model that captures the range of factors influencing the evolution of the fund and the Norwegian economy. We model expected and realised returns for equities and fixed income, alongside Norwegian economic growth, the exchange rate and petroleum revenues. The model is calibrated to capture the starting points for the fund and fiscal deficit described in this section, and allows us to quantitatively assess their future possible evolution.

3. Simulation model and calibration

In this section, we outline the simulation model we use to analyse the fund and fiscal spending. We rely significantly on previous work modelling asset prices and expected returns. In particular, we build on the simulation model of US fixed income and equity returns outlined in NBIM (2023). At the core of the model are macroeconomic trends and cycles, which are used as inputs for modelling the yield curve and equity prices. We extend this model to include the Norwegian economy, the fiscal budget, the USDNOK exchange rate, and petroleum revenues. We use the US economic variables and asset prices as proxies for the global economy and portfolio of the GPFG. Using the approach of NBIM (2022), we construct time-varying long-term expected returns on the fund, which are key for modelling the sustainability of the fund.

We describe these extensions below, and at the end of the section we describe the model calibration and initial values. For US macro and asset prices, we provide additional details in Appendix A. We simulate all variables in the model at a quarterly frequency.

Norwegian output and inflation

We first outline processes that characterise the Norwegian economy. We specify processes for real output growth and inflation in terms of persistent and transitory components, following the approach in NBIM (2023). When modelling output growth in Norway, we refer to mainland GDP growth, and this is reflected in the calibration described below. Log real output growth is denoted by z_t^{NO}, and its persistent and transitory components, denoted by $\tau_t^{z,NO}$ and a_t^{NO}, respectively:

$$z_t^{NO} = \tau_t^{z,NO} + a_t^{NO} \tag{1}$$

$$\tau_t^{z,NO} = \left(1 - \rho_z^{NO}\right) \mu_z^{NO} + \rho_z^{NO} \tau_{t-1}^{z,NO} + \rho_z^g \tau_{t-1}^z + \varepsilon_t^{z,NO} \tag{2}$$

$$a_t^{NO} = \rho_a^g a_{t-1} + \varepsilon_t^{a,NO}. \tag{3}$$

Norwegian inflation, π_t^{NO}, is also modelled as a sum of persistent and transitory components, $\tau_t^{\pi,NO}$ and c_t^{NO}:

$$\pi_t^{NO} = \tau_t^{\pi,NO} + c_t^{NO} \tag{4}$$

$$\tau_t^{\pi,NO} = \left(1 - \rho_\pi^{NO}\right) \mu_\pi^{NO} + \rho_\pi^{NO} \tau_{t-1}^{\pi,NO} + \rho_\pi^g \tau_{t-1}^\pi + \varepsilon_t^{\pi,NO} \tag{5}$$

$$c_t^{NO} = \rho_c^{NO} c_{t-1}^{NO} + \rho_c^g c_{t-1} + \varepsilon_t^{c,NO}. \tag{6}$$

Output growth and inflation in Norway are both directly linked to the US economy through the loadings on US components $\tau^z_{t-1}, \tau^\pi_{t-1}$ and a_{t-1}, c_{t-1} as well as through positive cross-country correlations between shocks in the persistent and transitory component processes.

We calibrate the parameters for the output growth and inflation processes to match moments from historical data. Table 1 shows simulated moments of real GDP growth and inflation in Norway, against their historical counterparts. For standard deviations and autocorrelations, we closely match the historical data. The table also shows cross-country correlations between the Norwegian macro variables and their US equivalents.[4] The long-term averages of output growth and inflation, μ^{NO}_z and μ^{NO}_π, are set at 1.7% and 2.2%, respectively. These values are informed by forecasts of long-term growth and inflation from Consensus Economics and Statistics Norway. Appendix B describes the calibration of US macro variables.

TABLE 1 Moments of Norwegian macro variables, historical vs. simulated

Variable	σ (%, Hist.)	σ (%, Sim.)	AC (Hist.)	AC (Sim.)	ρ (Hist.)	ρ (Sim.)
Real GDP Growth	2.2	1.9	0.75	0.81	0.52	0.55
Inflation	3.3	3.2	0.96	0.86	0.72	0.68

NOTE: σ and AC refer to annualised standard deviation and first-order autocorrelation, respectively. ρ refers to the correlation with the corresponding variable in the US. Observed moments are estimated using quarterly data for Norwegian mainland GDP and CPI excluding energy, over the period from Q1 1968 to Q4 2022. Data are sourced from Statistics Norway and Global Financial Data.

Norwegian fiscal expenditure and revenues

Based on the Norwegian macro processes, we model fiscal expenditures and revenues, from which we obtain fiscal deficits. At this stage, we deliberately describe a fiscal deficit that is not tied to expected returns on the fund. The initial value for the deficit is in line with the initial expected return on the fund, but in our analysis we allow for the possibility that they diverge. Later in the note, we use this separation of the deficit and expected returns to compare alternative rules for fund withdrawals. Based on the processes for output growth and inflation, we have a model of nominal GDP in Norway. The level of nominal GDP, denoted by Y_t, evolves as follows:

$$Y_t = Y_{t-1} exp\left(z^{NO}_t + \pi^{NO}_t\right). \tag{7}$$

We use the level of nominal GDP as an input into modelling the fiscal budget including government spending, S_t, and revenues, R_t. Both variables refer to the central government and exclude the oil sector, such that we model a 'non-oil' fiscal

[4] To generate correlations across Norway and the US, it is important that the trend components for output and inflation correlated across Norway and the US.

budget.[5] We model spending and revenues as a share of nominal GDP:

$$\frac{S_t}{Y_t} = \alpha^S + \varepsilon_t^S \tag{8}$$

$$\frac{R_t}{Y_t} = \alpha^R + \varepsilon_t^R. \tag{9}$$

α^S and α^R describe the steady-state ratios of spending and revenues to GDP, respectively.[6] ε_t^S and ε_t^R describe cyclical variation in spending and revenues. The calibration of the persistence of the cyclical components and their correlation with GDP growth can generate smooth counter-cyclical spending and pro-cyclical revenues, in line with historical experience.

The steady-state ratios are key parameters for describing the long-term behaviour of the fiscal budget. For our simulation analysis, we initially assume that the ratios of spending- and revenues-to-GDP are unchanged *on average* over all simulation horizons. This is equivalent to assuming that the fiscal deficit-to-GDP ratio, $(S_t - R_t)/Y_t$, does not change on average. We later relax this assumption in order to allow for alternative paths for the average fiscal deficit.

For fiscal expenditure and revenues, we calibrate the parameters for the S_t/Y_t and R_t/Y_t processes to match historical moments. For the calibration, we also target the moments of the deficit-to-GDP ratio, $(S_t - R_t)/Y_t$, and correlations of each ratio with nominal GDP growth. Table 2 shows the moments of simulated and historical data. We generate a relatively high degree of persistence in the ratios to GDP, and negative correlations between spending and deficit ratios and nominal GDP, in line with historical experience.

TABLE 2 Moments of revenues and expenditure, historical vs. simulated

Variable	σ (%, Hist.)	σ (%, Sim.)	AC (Hist.)	AC (Sim.)	ρ (Hist.)	ρ (Sim.)
S_t/Y_t	3.0	2.8	0.79	0.73	-0.53	-0.55
R_t/Y_t	1.9	1.4	0.90	0.93	0.14	0.16
$(S_t - R_t)/Y_t$	3.3	3.4	0.87	0.79	-0.55	-0.51

NOTE: σ and AC refer to annualised standard deviation and first-order autocorrelation, respectively. ρ is the correlation with nominal mainland GDP growth. Observed moments are estimated using annual data for revenues and expenditures excluding petroleum revenues over the period from 1985 to 2021. Data are sourced from Statistics Norway.

Yields and the exchange rate

Next, we outline our approach to modelling the Norwegian Krone (NOK) exchange rate. Since the fund is invested outside of Norway, the international value of the fund needs to be converted to NOK. We proxy a global portfolio using modelled US

[5] In the period between 1985 and 2021, the correlation between nominal mainland GDP growth and fiscal (non-oil) revenue growth is 0.74. This suggests it is reasonable to tie revenue growth closely to nominal GDP growth in the model.

[6] In an ideal case, the steady-state ratios should be modelled using the long-term component of the level of GDP, e.g., potential GDP. However, extracting the long-term component of the GDP level from equation (7) is not trivial and would have to be implemented using a filtering technique applied to Y_t. We avoid this step to keep the analysis simple and robust.

macro and asset prices, and therefore model a single exchange rate rather than multiple currencies. We model the USDNOK exchange rate as a function of yield curve differentials across the US and Norway, and a risk premium component.

We model the yield curve for Norway using the same specification as for the US, which we describe in more detail in Appendix A. NBIM (2023) provides detailed discussion of the US government bond yield curve model. The yield curve modelling approach is an implementation of the following yield identity, for the n-period US government bond yield, $y_t^{(n)}$:

$$y_t^{(n)} = i_t^* + \frac{1}{n} \sum_{j=0}^{n-1} E_t\left(\bar{i}_{t+j}\right) + tp_t^{(n)},$$

where i_t^* is the long-term nominal interest rate, $\bar{i}_t$ is a cyclical interest rate component defined relative to the long-term rate, and $tp^{(n)}$ is a maturity-specific term premium. The Norwegian government bond yield is denoted by $y_{t,NO}^{(n)}$, and its term premium component by $tp_{NO}^{(n)}$. We model the log USDNOK exchange rate based on US and Norway yield curves and a risk premium component:

$$p_t^{FX} = n\left(y_t^{(n)} - y_{t,NO}^{(n)}\right) - n\left(tp_t^{(n)} - tp_{t,NO}^{(n)}\right) - \beta^{fx} \sum_{i=1}^{n} \theta_t^{(i)}. \tag{10}$$

In line with the evidence in Lustig, Stathopoulos, and Verdelhan (2019) and Greenwood, Hanson, Stein, and Sunderam (2022), we can express exchange rates in terms of long-term government bond yields adjusted for term premiums.[7] This essentially adjusts yields for term premiums to uncover short-term interest rate expectations. As modelled in NBIM (2023), $\theta_t^{(i)}$ refers to the risk premium for a dividend strip of maturity i , which we use as a proxy for expected excess returns from holding NOK versus USD.[8] For additional flexibility in calibrating the volatility and correlations of the exchange rate, we include a loading β^{fx} that can be adjusted to guide the riskiness of the exchange rate.

Table 4 shows the simulated and historical moments for Norwegian yields and the exchange rate. In the table, we include the historical and simulated correlations between Norwegian and US yields. In general, we are able to capture yield and exchange rate dynamics, while also generating high cross-country correlations.

Oil price and petroleum revenues

As described earlier, proceeds from the production of oil and gas in Norway are accumulated in the GPFG over time. In our simulations of the fund, we need to account of these inflows by modelling the evolution of future petroleum revenues. We follow a similar approach to NBIM (2016), where we model an oil price that determines petroleum revenue inflows. Our modelling assumptions are deliberately simplistic, where in general we only aim to broadly capture the magnitudes and variability of revenues.

[7] Equation (10) holds for $n \to \infty$ and for $\beta^{fx} = 1$. However, to obtain an accurate approximation one does not need to model the components all the way to infinity.

[8] This modelling choice is motivated by internal research that suggests that the variation in the currency risk premiums is closely related to the variation in equity risk premium.

TABLE 3 Moments of Norwegian yields and USDNOK, historical vs. simulated

Variable	σ (%, Hist.)	σ (%, Sim.)	AC (Hist.)	AC (Sim.)	ρ (Hist.)	ρ (Sim.)
2Y Yield	3.7	2.0	0.996	0.92	0.76	0.69
5Y Yield	3.6	2.3	0.996	0.91	0.85	0.61
10Y Yield	3.5	2.2	0.997	0.93	0.94	0.61
USDNOK Return	11.4	11.4	0.04	0.13	-	-

NOTE: σ and AC refer to annualised standard deviation and first-order autocorrelation, respectively. ρ is the correlation with nominal US yield of the same maturity. Observed moments are estimated using quarterly data over the period from Q1 1960 to Q4 2022 for yields, and Q1 1971 to Q4 2022 for USDNOK. Data are sourced from FRED and Global Financial Data.

We model oil and gas revenues as a function of the oil price only. The real (log) price of oil, p_t^o is modelled as an autoregressive process that also includes the cyclical component of US growth, a_t, and an oil-market shock, ε_t^o, which we model as a supply shock:

$$p_t^o = (1 - \phi^o)\,\mu^o + \phi^o p_{t-1}^o + \phi^a a_t + \varepsilon_t^o \tag{11}$$

This implies that the real oil price is flat on average in the simulations. The oil shock, ε_t^o, is negatively correlated with growth and positively correlated with inflation, for both the US and Norway. We calibrate oil price growth to be weakly positively correlated with equity returns. Table 4 shows the simulated and realised moments.

TABLE 4 Moments of oil price changes, historical vs. simulated

Variable	σ (%, Hist.)	σ (%, Sim.)	AC (Hist.)	AC (Sim.)	ρ (Hist.)	ρ (Sim.)
Oil Price Changes	38.4	37.8	-0.13	-0.24	0.19	0.09

NOTE: σ and AC refer to annualised standard deviation and first-order autocorrelation, respectively. ρ is the correlation with US total equity returns. Observed moments are estimated using 3-month Brent crude oil futures prices over the period Q2 1988 to Q4 2022.

We specify the following process for government oil revenues, expressed in Norwegian Krone, where b_t and c_t are the production quantities and costs, respectively:

$$O_t = 0.85 \max\left(\left[b_t P_t^o P_t^{FX} - c_t\right], 0\right). \tag{12}$$

Petroleum revenues are determined by the level of production b_t, in billions of barrels, multiplied by the oil price and converted to Norwegian Krone based on the prevailing exchange rate, P_t^{FX}. Net income is obtained by subtracting costs, c_t, which are expressed in billion NOK. As in NBIM (2016), we bound O_t to be non-negative. For oil revenues, we assume an initial production level of one billion barrels per year and annual costs of 260 billion NOK.[9] The net government

[9] Production and cost levels are calibrated using figures from Norsk Petroleum for 2021. For simplicity, we use the oil-equivalent gas production values as inputs into a total production value.

revenues are set equal to 85% of the production value, which represents the effective tax rate on oil sector production, and revenue are bounded at zero.[10] We assume that these values decline to zero over a 30-year horizon.[11]

Realised and expected returns

Next, we define GPFG returns within the model and outline how we construct expected returns. We use the return generating model for the US from NBIM (2023), details of which are included in Appendix A. Summary statistics for equity and fixed income returns are included in Appendix B. In the model, equity and fixed income prices are a function of expectations of long-term growth and inflation, cyclical risk premiums, and interest rate cycles. Returns on these assets are used as a proxy for realised returns on the GPFG portfolio.

We denote simple returns on US fixed income and equity as r_t^{FI} and r_t^{EQ}, respectively, where equity returns include dividends. We assume a 70-30% equity-bond portfolio where the weights are fixed over time.[12] The nominal portfolio return, r_t^P, is given by:

$$r_t^P = \left(w^{EQ}\left(1 + r_t^{EQ}\right) + w^{FI}\left(1 + r_t^{FI}\right)\right)\left(1 + r_t^{FX}\right) - 1. \quad (13)$$

where $w^{EQ} = 0.70$ and $w^{FI} = 0.30$. Asset returns are modelled in US dollars, therefore r_t^P also incorporates changes in the exchange rate, where r_t^{FX} is the percentage change in the USDNOK rate. The structure of our model allows us to construct expected returns on equity and fixed income following the framework described in NBIM (2022). We model long-term expected equity returns, $E_t\left(r_{t,\infty}^{EQ}\right)$, in terms of 'carry' and 'growth' components:

$$E_t\left(r_{t,\infty}^{EQ}\right) \approx \underbrace{E_t\left(D_{t+1}\right)/P_t}_{\text{Carry}} + \underbrace{G_{t,\infty}}_{\text{Growth}}, \quad (14)$$

The carry component is the expected dividend next period, scaled by the equity index price. The growth component of expected returns is the sum of dividend growth rates across all future periods, given by:

$$G_{t,\infty} = \sum_{n=2}^{\infty} w_t^{(n-1)} E_t \Delta d_{t+n}, \quad (15)$$

where Δd_{t+n} refers to nominal dividend growth between $t+n-1$ and $t+n$. The weights $w_t^{(n)}$ in equation (15) are defined as the present value of the dividend payout at time $t+n$ divided by the present value of all future dividends, which corresponds to the current equity index price P_t. We define the expected real return on the fund in terms of expected returns across asset classes and

[10] This effective tax rate is estimated based on the historical relationship between net government petroleum revenues and the market value of production.

[11] Given high initial values starting in 2022, we impose a path where production and cost levels halve at around the 3-year horizon, and are three-quarters below their initial values around the 10-year horizon.

[12] We assume that portfolio weights are continuously rebalanced each quarter. Allowing weights to drift to an extent would not affect any results presented in this note.

subtracting expected inflation:

$$E_t\left(r_{t,\infty}^R\right) = w^{EQ} E_t\left(r_{t,\infty}^{EQ}\right) + w^{FI} y_t^{(n)} - E_t\left(\pi_{t,\infty}\right), \tag{16}$$

where $E_t\left(\pi_{t,\infty}\right)$ refers to the expected inflation across all horizons, for the US. For the fixed income portfolio, we use the n-maturity yield $y_t^{(n)}$ as a proxy for the long-term expected return.[13] Table 5 shows the simulated and historical moments for the different expected return components. The volatility of simulated expected returns on the fund is higher than in the historical data. A key reason for this is that historical volatility of expected returns, which are very persistent, is estimated in a relatively short sample period.

TABLE 5 Moments of expected return and inflation, historical vs. simulated

Variable	σ (%, Hist.)	σ (%, Sim.)	AC (Hist.)	AC (Sim.)
Expected Real Return	0.8	1.4	0.97	0.97
Expected Inflation	1.9	1.4	0.99	0.998

NOTE: σ and AC refer to annualised standard deviation and first-order autocorrelation, respectively. Historical expected return series are taken from NBIM (2022), over the period from Q1 2003 to Q4 2022.

Simulation initial values

We set the initial values for the simulation model based on values at the end of Q4 2022, where applicable. Table 6 shows the initial values for variables in the Norwegian economy, financial market variables and the expected return for the fund. We assume an initial fund size of 12,500 billion NOK, and set the initial value of withdrawals equal to 375 billion NOK. We set mainland GDP equal to 3,750 billion NOK, which implies an initial deficit-to-GDP equal to 10%.[14]

We set the initial shares of fiscal expenditure and revenues relative to nominal GDP to 46% and 36%, respectively. These values are aligned with those presented earlier in Section 2, where the fund withdrawal is around 20% of fiscal expenditure. The values for fiscal expenditure and revenues are broadly in line with the figures from the Norwegian National Budget for 2023.

For financial markets, we calibrate the initial yield curves, the dividend-price ratio, and dividend growth, to match the expected return estimates in NBIM (2022). These values are set in line with weighted expected return estimates for G4 markets, implying an initial expected real return on the fund of 3%, in line with the most recent value presented earlier in Figure 3.[15] We also set the long-term

[13] We use log growth rates and yields as inputs into our expected return calculations. These need to be converted in order to obtain expected simple returns, which involves adding a small adjustment to both equities and fixed income estimates.

[14] These values are higher than estimates for end-2022, but this better aligns the budget deficit with the real expected return multiplied by the fund value. This simplifies later comparisons between withdrawals based on either expected returns or the Norwegian business cycle.

[15] We use market capitalisation weights for equities and GDP weights for fixed income to combine estimates across the US, euro area, UK and Japan. We set the initial value of long-term inflation expectations equal to 2%.

TABLE 6 Simulation initial values

Variable	Value
Panel A. Norway	
Fund Value (bn NOK)	12,500
Mainland Norway GDP (bn NOK)	3,900
Deficit-to-GDP (%)	10.0
Expenditure Share of GDP (%)	46
Revenue Share of GDP (%)	36
Gov. Expenditure (bn NOK)	1,794
Gov. Revenues (bn NOK)	1,404
Oil Revenues (bn NOK)	958
Panel B. Financial Markets	
10Y Yield (G4, %)	3.1
10Y Yield (NO, %)	3.1
Dividend-Price Ratio (G4, %)	2.5
Dividend Growth (G4, %)	3.3
Expected Inflation (G4, %)	2.0
70-30 Expected Real Return (%)	3.0
Oil Price (USD)	90
USDNOK	9.8

NOTE: Values in Panel A are informed by figures from the Norwegian Ministry of Finance and Statistics Norway.

averages of inflation and growth in line with weighted averages of survey expectations for G4 markets from Consensus Economics, and for simplicity ensure that *on average* the expected return on the fund stays around the same level at the starting value.[16,17] The initial value of the USDNOK exchange rate is equal to 9.8, and the oil price is set equal to 90 US dollars.[18] This oil price corresponds to initial annual net government oil revenues of 958 billion NOK.

We use the simulation model to generate macroeconomic variables and asset returns over horizons up to 30 years. From the initial values described above, we simulate 10,000 alternative paths, which are used to track distributions of the value of the fund alongside other variables.

[16] The model dynamics are calibrated to US data, while the starting levels and long-term averages are set in line with G4 markets. We need to use G4 data in order to match the estimate for the expected return on the fund. Calibrating dynamics based on US data is still representative to the extent that there is a high degree of co-movement across the G4 markets. The US is the largest financial market, and can account for large proportion of variation in asset returns.

[17] There remains a wide distribution of possible paths for expected returns. We align long-term averages with initial values to simplify interpretation of results. We discuss the effects of different long-term averages on long-horizon returns in detail in NBIM (2023).

[18] As we model oil and gas revenues as a function of the oil price only, we use an adjusted oil price for the initial value that combines natural gas and oil prices.

4. Comparing sustainable and cyclical fund withdrawals

As outlined in Section 2, the spending rule outlines two main considerations for withdrawals from the fund. First, transfers from the fund should follow the expected real return on the fund over time, in order to preserve the value of the fund in expectation over the long-term. Second, withdrawals from the fund should be used to accommodate cyclical fiscal spending.

In this section, we use the simulation model to understand how the objectives of sustainability and cyclicality of fund withdrawals interact. To do this, we consider each objective in isolation, and explore the implications for withdrawals and the value of the fund over short and long horizons.

Defining sustainable and cyclical fund withdrawals

Using the model described in in the previous section, we are able to define alternative expressions for withdrawals from the fund. The evolution of the value of the fund in NOK, V_t, is determined by the portfolio return, r_t^P, as well as inflows and outflows. Initially, we conduct our analysis excluding petroleum revenue inflows, such that the fund evolves as follows:

$$V_t = \max\left(V_{t-1}\left(1 + r_t^P\right) - W_t, 0\right), \tag{17}$$

where W_t is the withdrawal from the fund. The fund process allows us to define a spending rule that ensures the fund is preserved in real terms, on average.[19] To do this, withdrawals from the fund need to be set in line with the expected real return on the fund, multiplied by the value of the fund.[20] We define withdrawals based on fund preservation as W_t^1, which we refer to as the 'Preservation Rule':

$$W_t^1 = E_{t-1}\left(r_{t-1,\infty}^R\right) V_{t-1}, \tag{18}$$

where $E_{t-1}\left(r_{t-1,\infty}^R\right)$ is the expected real return on the fund, as defined in equation (16).[21] This rule highlights that expected returns and the value of the fund are the only considerations that should be taken into account when strictly targeting preservation of the fund. As a reminder, under the fiscal policy framework, withdrawals from the fund are equal to the fiscal deficit. The Preservation Rule therefore also implies behaviour of deficits that is entirely determined by expected and realised asset returns on the fund.

Next, we derive a rule for fund withdrawals that is determined only by spending requirements in the Norwegian economy. We use the processes for government

[19] This definition of preservation does not preclude the possibility of significant depletion of the fund.

[20] Specifically, this refers to the expected simple average return. Dybvig and Qin (2021); Campbell and Sigalov (2021); Mork, Trønnes, and Bjerketvedt (2022) argue that a geometric average return may be more appropriate for preserving the fund, due to a 'volatility drag' effect. This effect tends to be meaningful only at very long horizons in our model, and is further attenuated if the expected return estimate does not incorporate additional sources of returns, such as portfolio inflows.

[21] Following NBIM (2022), we use a long-term expected return estimate. Strictly, the one-period-ahead expected return should be used to stabilise the fund on average. In our simulations, however, the long-term expected return provides a good approximation for preserving the fund on average and is more practical from a fiscal budgeting standpoint due to its lower volatility. We set current period withdrawals based on expected returns and fund values in the previous period.

revenues and expenditure outlined in the previous section to define W_t^2. We refer to this approach as the 'Business Cycle Rule':

$$W_t^2 = \left(\alpha^S - \alpha^R\right) Y_t + \left(\varepsilon_t^S - \varepsilon_t^R\right) Y_t. \tag{19}$$

This expression follows from combining equations (8) and (9), where withdrawals are equal to fiscal expenditure minus revenues.[22] $\left(\alpha^S - \alpha^R\right) Y_t$, represents the long-term level of fund withdrawals or fiscal deficits, equal to 10% of GDP on average over all horizons.[23] $\left(\varepsilon_t^S - \varepsilon_t^R\right) Y_t$, captures short-term or cyclical variation due to fiscal shocks. Given counter-cyclical spending and pro-cyclical revenue, this component of withdrawals is strongly counter-cyclical.

An initial comparison of W_t^1 and W_t^2 highlights differences between the two objectives of the fiscal spending rule. A key difference is that the fund value features in the Preservation rule and not in the Business Cycle rule. In addition, withdrawals under the two rules differ in terms of their average path over time. The path for the Preservation rule is flat when excluding petroleum revenues. When spending the expected real return, multiplied by the fund value, both withdrawals and the fund value stay flat on average (in real terms). On average, for the Business Cycle rule, the deficit-to-GDP ratio is flat, but in NOK terms the deficit will increases each year. This is because Y_t increases on average, in line with nominal GDP growth, implying that the level of withdrawals can diverge across the two rules over the long term. In addition, alignment across the two rules would require a high correlation between the fund value, expected returns, and the Norwegian economy. Next, we use the simulation model to quantitatively assess these effects.

Simulating withdrawals and the evolution of the fund

We use the simulation model to illustrate how fund withdrawals and the evolution of the value of GPFG differ under the two rules. The simulations allow us to quantitatively explore the implications of different spending policies over the short and long term. First, we look at the short-term properties of withdrawals under the Preservation and Business Cycle rules. Figure 4 Panel (a) shows how withdrawals/deficits compare, plotting W_t^1 and W_t^2 in the first year of the simulation against one another. We show the withdrawals at a one-year horizon to focus on cyclical properties, so that this analysis is not affected by the divergence in withdrawal levels over longer horizons.

The correlation between the two alternatives is positive but not high, at around 0.20. This implies there are many instances where, under the Business Cycle rule, it is necessary to increase fiscal deficits, but the withdrawals defined by the Preservation rule would imply that deficits need to be below this level. In other words, the objective of preserving the fund would be at odds with the objective of accommodating cyclical spending needs. In the scatter plot, all outcomes below

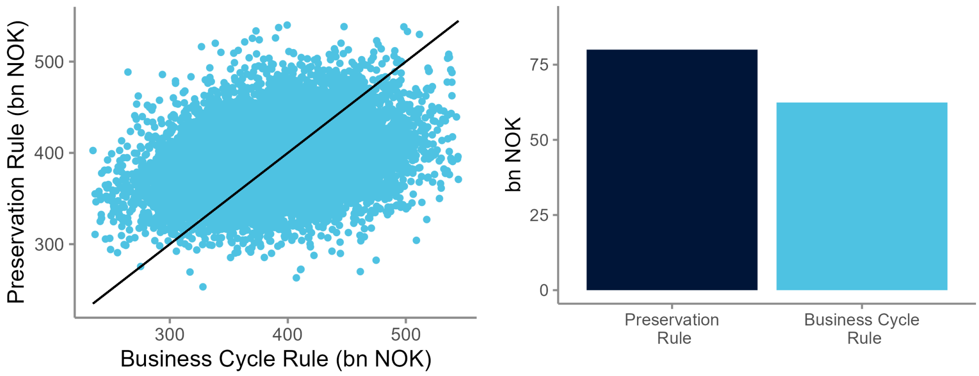
[22] We exclude the use of debt in financing deficits. The Government Pension Fund Act states in Section 7(2) that 'Central government shall not fund central government budget expenditure by borrowing as long as there is capital in the Government Pension Fund Global.'

[23] For modelling convenience, the α parameters are multiplied by Y_t. As a result, this component of W_t^2 will also include some cyclical variation. If necessary, we could separately identify a long-term component of the level of GDP.

FIGURE 4 Short-term properties of fund withdrawals

(A) Annual fund withdrawals: Preservation vs. Business cycle rule

(B) Average absolute year-on-year change in fund withdrawals



NOTE: Panel (a) plots fund withdrawals over the first year of simulations for alternative rules. Panel (b) shows the average absolute change in yearly withdrawals from the fund, in billion NOK, for both rules, over the first three years of the simulation.

the 45-degree line indicate paths where the Preservation rule would prescribe a smaller deficit than required by the Business Cycle rule. In these circumstances, it would not be possible to define withdrawals in a way that is both sustainable and sufficiently cyclical.

The positive correlation between withdrawals arises in part due to the negative correlation between the NOK exchange rate and Norwegian GDP growth. The currency tends to depreciate when GDP growth is weak, and the fiscal deficit is high, which increases the NOK value of the fund at these times. Indeed, the Norwegian Krone depreciated during past episodes of increased fiscal spending in Norway, such as the 2008/09 financial crisis and COVID-19 pandemic. This is not a perfect negative correlation, however. This divergence can be accounted for through lower correlations of the Norwegian fiscal deficit with asset returns and expected returns.[24]

The two rules also imply different levels of persistence in fund withdrawals/deficits over time. Figure 4 Panel (b) shows the average absolute year-on-year change in fund withdrawals for the two rules. The Preservation rule inherits volatility from changes in the value of the fund. In addition, expected returns vary over time, leading to further changes in fund withdrawals each year. As a result, the average annual change in fund withdrawals is higher for the Preservation rule. The Business Cycle rule provides a benchmark against which to compare this variability. Based on the calibration of the fiscal budget, that matches historical persistence in spending and revenues, the average annual change is around ten percent lower than the Preservation rule.

Next, we examine the implications of the two rules for the value of the fund over

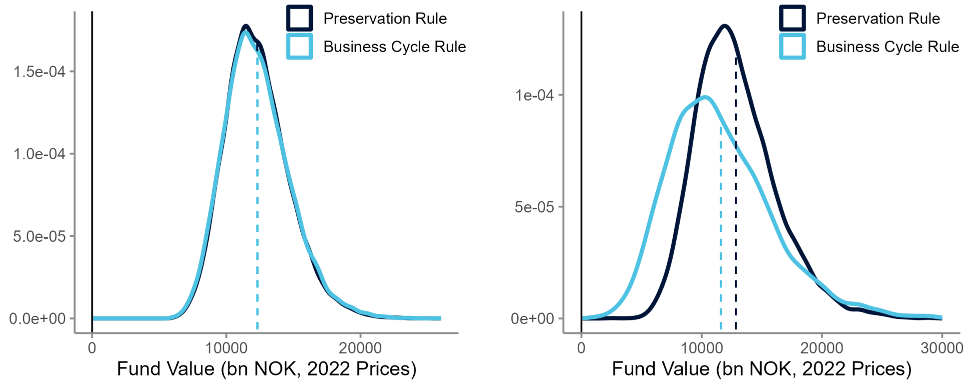
[24] Variation in withdrawals under the Preservation rule arises from both changes to the value of the fund and variation in long-term expected returns. To the extent that the expected return reflects long-term growth expectations outside of Norway, we wouldn't expect short-term cyclical dynamics in Norway to strongly co-move with expected returns. Similarly, the expected return contains risk premium components that also do not have a clear association with the fiscal budget.

different horizons. Figure 5 shows the distributions at the 3- and 20-year horizons, expressed in real terms.[25] Panel (a) shows that, over the short term, the distribution of fund values is nearly identical under the two rules. This result occurs despite the differences in withdrawal patterns across the two rules.

FIGURE 5 Distributions of fund values at 3- and 20-year horizons for alternative spending rules

(A) Fund value (bn NOK, 2022 prices) - 3-year horizon

(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

This result holds for most rules that spend a relatively small proportion of the total fund value. The variability of expected returns and Norwegian economic variables would have to be implausibly high to generate meaningfully different distributions at short horizons. The differences in withdrawals between the rules are small in comparison to the variation in the fund value due to asset prices and the exchange rate. Over shorter horizons, therefore, the cyclical spending and fund preservation objectives do not meaningfully conflict. Over horizons of a few years, it is possible to spend counter-cyclically from the fund without impacting the prospects for preservation of the fund.

The trade-off between sustainable and cyclical withdrawals is more apparent at longer horizons, however. Figure 5 Panel (b) shows the distributions of fund values at the 20-year horizon. The Preservation rule provides the benchmark for the long-term distribution of the fund when spending sustainably. For this distribution, the average real value is aligned with the initial value of the fund, at 12,500 billion NOK. The left tail of the distribution is also relatively bounded. The Preservation rule spends a percentage of the fund value, meaning that when the fund is depleted, withdrawals are reduced proportionally.

The distribution of fund values based on the Business Cycle rule lies to the left of the Preservation rule distribution. The position of the Business Cycle rule distribution partly reflects the higher average withdrawals for cyclical spending. As shown in equation (19), the level of withdrawals increases on average over time in line with nominal GDP (given the deficit-to-GDP ratio is flat on average). This higher

[25] We deflate the value of the fund using the simulated Norwegian price index.

level of spending means the average value of the fund is lower than under the Preservation rule. It also follows that the average deficit-to-GDP ratio under the Preservation rule *declines* over time, to around 7.5% when excluding oil revenues from the analysis. This implies that total spending declines relative to GDP, by around two and a half percentage points over 20 years to 43.5%.

The pattern of withdrawals under the Business Cycle rule also leads to a higher probability of depleting the fund. The probability of the real fund value being 50% lower at the 20-year horizon is below half a percent for the Preservation rule, and around 5% for the Business Cycle rule. This divergence increases with horizon. For example, at the 30-year horizon, the probability remains below 1% for the Preservation Rule, but increases to near to 30% for the Business Cycle rule.

At long horizons, repeated periods of cyclical spending mean that the fund is not preserved in expectation, and the probability of depletion increases. The longer-term distributions highlight the trade-off between sustainable and cyclical spending. We include an additional exercise that depicts this trade-off in Appendix C. We combine the Preservation Rule with the cyclical component of the Business Cycle rule. While anchoring withdrawals to long-term expected returns helps preserve the fund, variation in the fund and expected returns still generates deviations relative to the Business Cycle rule.

Simulating rules with petroleum revenue inflows

So far, for simplicity, we have excluded petroleum revenues and fund inflows from our analysis. Next, we extend the simulations to include these inflows into the fund. We repeat our simulations where the evolution of the fund includes oil revenue inflows, O_t, expressed in NOK terms:

$$V_t = V_{t-1}\left(1 + r_t^P\right) + O_t - W_t. \tag{20}$$

We define the Preservation and Business Cycle rules in the same way as earlier. In particular, we continue to define the Preservation rule as spending the expected return on the investment portfolio. As in reality, the rule does not include expected future petroleum revenues when defining expected returns. This implies that as revenues are paid into the fund, the real value will increase over time on average.[26] In the earlier periods of the simulations, this effect will be particularly pronounced given recent increases in oil revenues to record highs. This also means our simulations are sensitive to assumptions around whether these high revenues persist. We take a conservative approach, assuming that revenues stay at high levels in the first year, and then revert closer to historical levels in subsequent years.

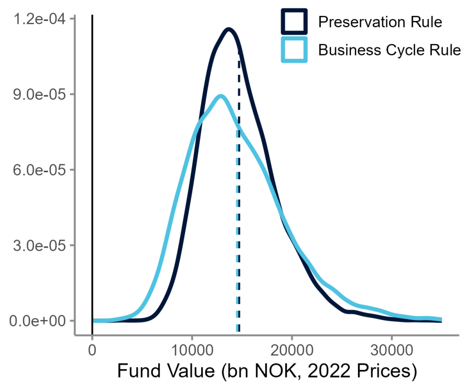
Figure 6 Panel (a) shows the 20-year distributions of fund values when including oil revenues. Relative to the analysis excluding oil, both distributions shift to the right, due to the cumulation of petroleum revenue inflows. This leads to the two rules withdrawing similar amounts from the fund, on average, over the long-term. When

[26] This is consistent with the aim of preserving the current value of the fund, which is the cumulation of past petroleum revenues.

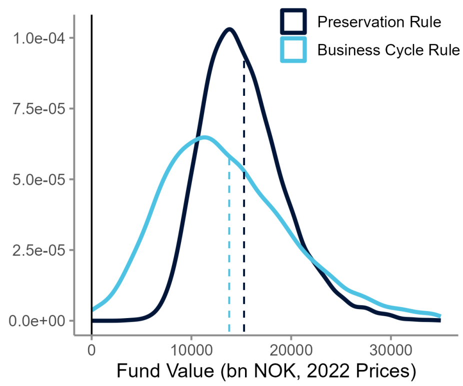
spending the expected real return on the portfolio, oil revenues cumulate over time, and the fund expected return is multiplied by a higher value fund on average. When including inflows, the average deficit-to-GDP ratio under the Preservation rule declines to around 9% at the 20-year horizon. This results in similar average values of the fund across the two rules. However, the distribution of the fund remains wider for the Business Cycle rule, and there is a higher probability of a decline in the real fund value. In this sense, the conclusions from the analysis excluding oil are unchanged.

FIGURE 6 Distributions of fund values at the 20- and 30-year horizons for alternative spending rules - including oil revenues

(A) Fund value (bn NOK, 2022 prices) - 20-year horizon



(B) Fund value (bn NOK, 2022 prices) - 30-year horizon



NOTE: Dashed lines depict mean values. Real values are determined using the simulated price index for Norway

As the horizon extends, the distributions increasingly resemble those presented earlier excluding oil. As oil production eventually declines, petroleum revenues become a less important factor determining the outcomes for the fund. Figure 6 Panel (b) shows the distributions at the 30-year horizon. At this horizon, the average value of the fund under the Business Cycle rule falls below that for the Preservation rule. Similar to the results excluding oil revenues, spending from the fund under the Business Cycle rule grows in line with GDP, and eventually the average fund value declines relative to the Preservation rule.

5. Alternative approaches to setting withdrawals

In this section, we consider alternative rules for guiding withdrawals from the fund. We first consider a commonly proposed rule that spends the expected cash flows from the fund. We then look at a rule that targets 'smoothed' withdrawals. Both of these rules share similarities with the Preservation rule, in particular by linking withdrawals directly to the value of the fund. We then outline an 'Adjusted Business Cycle' (ABC) rule, that links withdrawals to GDP in line with the Business Cycle rule. The ABC rule focuses on accommodating cyclical spending requirements, but slowly reduces the withdrawal amounts over time. For all the analysis in this section, we simulate the fund including oil revenue inflows.

We use the results from the previous section to benchmark different rules. The extent to which a rule is able to accommodate cyclical spending requirements is evaluated relative to the benchmark provided by the Business Cycle rule. The more closely withdrawals patterns match the Business Cycle rule, the better the cyclical properties of a given rule. Similarly, the long-term distribution of the value of the fund under the Preservation rule provides a benchmark against which to evaluate the sustainability of a given rule. To the extent a rule generates a similar or lower probability of depleting the fund, a rule performs well in terms of preservation.

Cash flow-based rules

We use the model to explore a rule that defines withdrawals according to the cash flows that the portfolio generates. These 'cash flow' rules have regularly proposed in the investment industry at different points in time, in particular for different types of endowments. It is often argued that these rules are attractive since they reduce the variability in withdrawals compared to spending based on expected returns. In addition, the rule may be attractive given that withdrawals are based on realised cash flows from investments, rather than relying on market values of portfolios. These arguments have been made in the context of GPFG by Holden (2022), and a general overview of cash flow spending is provided in Garland (2019).

In our model, we approximate a cash flow rule in terms of the expected dividend-price ratio of the equity portfolio, and the yield on the fixed income portfolio:

$$W_t^{CF} = \left(w_{EQ} E_t \left(\frac{D_{t+1}}{P_t} \right) + w_{FI} y_t^{(n)} \right) V_{t-1}. \tag{21}$$

Here, the product of the dividend-price ratio and the equity value of the fund obtains the expected dividends from the fund. We approximate coupons on the fixed income portfolio using its nominal yield.[27]

This rule closely resembles the specification for the Preservation rule, where it also includes the value of the fund for defining withdrawals. The main difference, however, is that instead of the long-term expected real return on equity, the rule is based on the dividend-price ratio. As outlined in equation (14), the long-term expected return can be expressed as the sum of the dividend-price ratio and expected cash flow growth. Given positive long-term cash flow growth, the dividend-price ratio will be lower than total expected equity returns. The implies that the Cash flow rule will spend a lower amount that under the Preservation rule. It is well-known, however, that aggregate dividends are relatively smooth, especially compared to aggregate equity prices. Based on this, the Cash Flow rule likely provides a more stable stream of withdrawals from the fund.

Figure 7 Panel (a) shows the short-term properties of the Cash Flow rule. The chart shows withdrawals at the one-year horizon for the Cash Flow rule plotted against

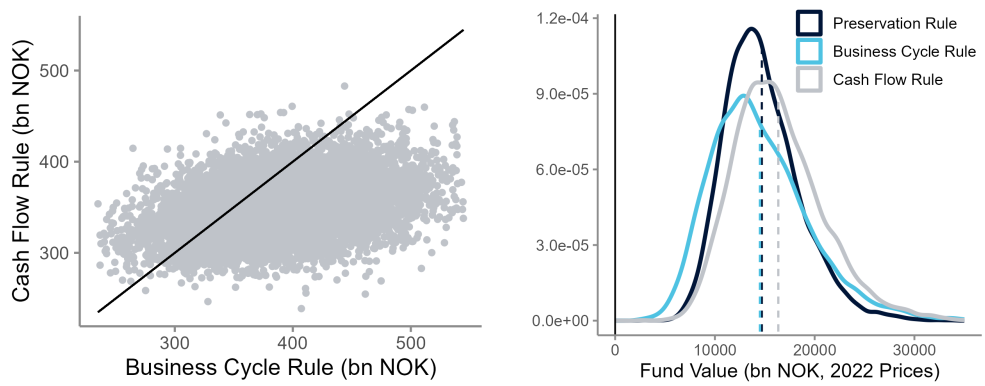
[27] For the implementation of this rule, we do not adjust for expected inflation. The dividend-price ratio is historically lower than the nominal expected return by a margin greater than expected inflation. This lower level means that it is in a region where the expected fund value is preserved in real terms even when using nominal cash flows.

withdrawals based on the Business Cycle rule.

FIGURE 7 Comparison of withdrawals and fund values under the Cash Flow Rule

(A) Annual fund withdrawals: Cash flow vs. Business Cycle rule

(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Panel (a) plots fund withdrawals over the first year of simulations for alternative rules. In Panel (b), dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

Similar to the results when spending based on expected returns, there is a positive correlation between these two rules.[28] Compared to earlier, however, the level of withdrawals under the Cash Flow rule is lower across all simulations. As a result, a significant proportion of the points lie below the 45-degree line. This implies that, for almost all states of the Norwegian economy, the required fiscal deficit under the Business Cycle rule would not be covered by spending the cash flows from the fund. The gap is largest in situations when the deficit under the Business Cycle rule is high. This implies that cash flows from the fund do not increase enough to accommodate downturns in the Norwegian economy. The average deficit-to-GDP ratio based on the Cash Flow rule decreases to near 8% across all simulation horizons, implying that total fiscal expenditure relative to GDP declines by around 2 percentage points.

Figure 7 Panel (b) compares the 20-year distributions fund values for the Cash Flow rule against the other rules. Since the Cash flow rule implies a relatively sharp reduction in deficits, this leads to higher values of the fund over the long term. These lower withdrawals shift the distribution of fund values to the right of both the Preservation and Business Cycle rules. This leads to a large reduction in the likelihood of depleting the fund, and the higher mean value implies that the fund is expected to grow over the long-term. Overall, the Cash Flow rule is able to meet the objective of fund preservation, but withdrawals based on this rule imply smaller fiscal deficits, and these deficits do not align with the Norwegian business cycle.

Smooth spending 'Tobin' rules

Next, we examine a rule that attempts to reduce variability in spending by 'smoothing' withdrawals. These types of spending rules combine withdrawals in

[28] In line with our priors, the Cash Flow rule delivers smoother spending in terms of year-on-year changes compared to the other rules.

recent periods with a target level of withdrawals, often set in line with market-based measures. One commonly-used version amongst endowments combines withdrawals in the previous period with a 'sustainable' rate multiplied by the fund value. This approach, often referred to as a 'Tobin' rule, can be implemented in our model as follows:

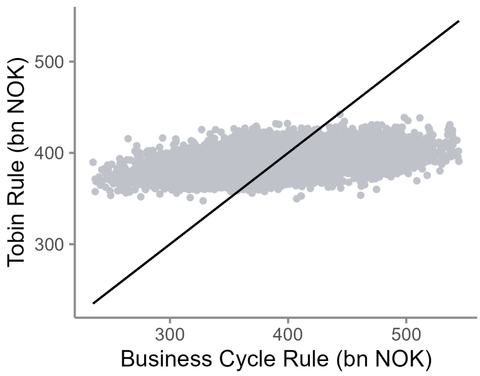
$$W_t^S = \phi \left(W_{t-1}^S (1 + \pi_t^{NO}) \right) + (1 - \phi) E_{t-1} \left(r_t^R \right) V_{t-1}. \tag{22}$$

Here, the withdrawal amount W_t^S is determined by a weighted average of last period's withdrawal adjusted for inflation (in Norway) and the expected real return on the fund. The weight, ϕ, determines the balance between the two components in influencing withdrawals. This approach is designed to lower the variability of withdrawals and reduce instances of sharp adjustments when the value of the fund changes. A related approach spends in line with a moving average of the fund value, and we include analysis of this approach in Appendix D.

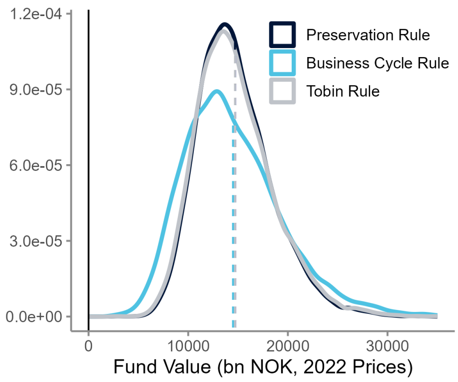
Figure 8 Panel (a) shows the short-term properties of withdrawals based on the Tobin rule relative to the Business Cycle rule. For this analysis, we set $\phi = 0.8$, such that a higher weight is placed on the recent level of withdrawals from the fund.[29]

FIGURE 8 Comparison of withdrawals and fund values under the Tobin Rule

(A) Annual fund withdrawals: Tobin vs. Business Cycle rule



(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Panel (a) plots fund withdrawals over the first year of simulations for alternative rules. In Panel (b), dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

The scatter plot shows that withdrawals do not align well with cyclical spending based on the Business Cycle rule. There are many simulations where withdrawals based on the Tobin rule lie below the 45-degree line, indicating a shortfall relative to the necessary fiscal spending. While the rule adds stability in the level of spending, it is not guided by developments in the Norwegian economy. This means that there is little co-movement between withdrawals under the Tobin and Business Cycle rules. Figure 8 Panel (b) shows the distribution of fund values at the 20-year horizon. The distribution under the Tobin rule is very similar to the distribution for the Preservation rule, and the rule generates similar expected

[29] We calibrate the rule in line with weights often used by US university endowments.

values and probabilities of depleting the fund. The inclusion of expected returns in the Tobin rule means its withdrawals are highly correlated with those under the Preservation rule. The persistence in spending means that when expected returns decline, withdrawals do not decline as much under the Tobin rule. This is also the case when expected returns increase, where the Tobin rule under-spends relative to the change in expected returns. These effects offset to produce a distribution of fund values that is close to the Preservation rule.

Adjusted Business Cycle rule

The analysis so far has shown the difficulty in reconciling cyclical spending requirements with spending in line with expected returns or portfolio cash flows. In general, linking withdrawals to fund values and expected returns means that they are unlikely to align with the Norwegian economic cycle. Based on this, we next consider rule that does not directly rely on fund's value or expected returns.

We outline a rule that aims to accommodate short-term cyclical requirements, while also improving the long-term distribution of the fund. Fund withdrawals are directly determined by fiscal processes and the Norwegian economy, similar to the Business Cycle rule presented earlier. A difference, however, is that the rule aims to reduce the fiscal deficit on average, where the deficit-to-GDP was previously assumed to be flat. We refer to this rule as the 'adjusted' Business Cycle - or 'ABC' rule. The withdrawal amount, W_t^{ABC}, is defined as follows:

$$W_t^{ABC} = \left(\alpha^S - k_t - \alpha^R\right) Y_t + \left(\varepsilon_t^S - \varepsilon_t^R\right) Y_t. \tag{23}$$

This spending rule closely resembles the Business Cycle rule described by equation (19), with an extra term, k_t. This 'adjustment' term subtracts from the steady-state value of spending relative to GDP, gradually reducing the fiscal deficit over the long-term. For illustration, we define k_t such that in each period the long-term target for the deficit-to-GDP ratio is reduced as follows:

$$k_t = \gamma + \beta\sqrt{h}\left(\frac{S_{t-1}}{Y_{t-1}} - \frac{\bar{S}}{\bar{Y}}\right), \tag{24}$$

where h is the number of elapsed periods from today. $\frac{\bar{S}}{\bar{Y}}$ is the target spending-to-GDP ratio, which is set below today's level. The degree of adjustment depends primarily on where the latest value of $\frac{S_t}{Y_t}$ is relative to the target. We assume that the ratio of revenues to GDP does not change on average. The target spending-to-GDP ratio therefore maps directly to a target deficit-to-GDP ratio.

We use two parameters to guide the degree of adjustment in the fiscal budget to the target level. γ is a fixed adjustment made each period, and β determines required adjustment based on the distance from the target spending. When there is a greater difference between the most recent and target spending, and a larger reduction in spending is required. $\sqrt{h}$ increases over time, meaning that larger adjustments are made to reach the target level as time passes. The rule continues to include the cyclical variation in deficits, determined by $\left(\varepsilon_t^S - \varepsilon_t^R\right)$.

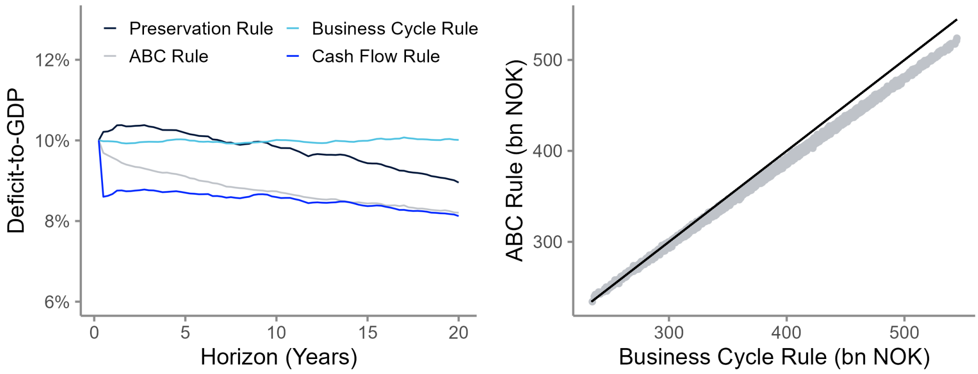
While the adjustment term in equation (24) may appear complex, this expression is only needed to generate a representation of spending adjustments within the model. The deficit reduction term can be defined in many ways, and we leave a detailed exploration of different specifications for future work. The key component of the ABC rule is the targeted reduction in the deficit-to-GDP ratio. Rather than tying spending from the fund to expected returns, the rule takes a deficit-to-GDP perspective. This is a standard approach in discussions around fiscal spending, where usually the focus is on the implications for government debt. The main mechanisms of the ABC rule could be captured in any fiscal framework that sets a target withdrawal amount that decreases as a proportion of Norwegian GDP.

Figure 9 Panel (a) shows the average deficit-to-GDP ratio for the ABC rule alongside other spending rules. As discussed earlier, this ratio is flat on average for the Business Cycle rule. For the Preservation rule, the ratio initially increases as petroleum revenue inflows lead the fund to grow on average. The ratio eventually declines, however, as Norwegian GDP grows and petroleum revenue inflows decline. The ABC rule on average adjusts gradually downward, where we calibrate the deficit to decrease to around 8% on average over 20 years.[30]

FIGURE 9 Deficit-to-GDP ratios and withdrawals under the ABC Rule

(A) Average deficit-to-GDP by horizon across spending rules

(B) Annual fund withdrawals: ABC vs. Business Cycle rule



NOTE: Panel (a) plots average deficit-reduction paths implied by alternative rules. In Panel (b), we show fund withdrawals over the first year of simulations for alternative rules. Real values are determined using the simulated price index for Norway.

Figure 9 Panel (b) shows how withdrawals under the ABC rule compare to the Business Cycle rule. Given that the ABC rule directly incorporates cyclical spending requirements, withdrawals are highly correlated with the Business Cycle withdrawals. Since the rule continuously attempts to reduce the fiscal deficit, the majority of withdrawals lie slightly below the 45-degree line. The reductions relative to the Business Cycle rule are limited, however. Reductions are more visible in scenarios where the Business Cycle rule draws more heavily on the fund, and the deficit as a proportion of GDP is high. In these scenarios, the ratio of spending to GDP will be higher and the ABC will adjust the long-term target for the deficit by a larger amount. These adjustments are still relatively limited, however. In principle, the rule could be adjusted to implement spending reductions more

[30] This is achieved by setting γ equal to 0.003 and β equal to 0.08.

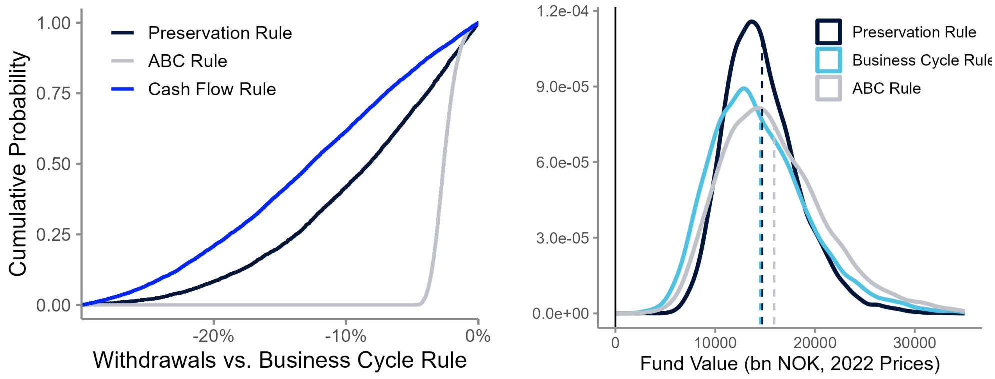
heavily during upturns in the Norwegian economy. Our version of the ABC rule is kept simple in order to illustrate its effects. We leave exploration of a more sophisticated rule and its implementation for future analysis.[31]

In Figure 10 Panel (a), we compare how well the ABC and other rules align with the Business Cycle rule. The chart shows the cumulative probability of a shortfall in withdrawals relative to the Business Cycle rule. The chart measures how often a rule generates withdrawals below the 45-degree line in the scatter charts shown so far.

FIGURE 10 Long-term distribution of fund values and spending adjustments

(A) Cumulative probability function for withdrawals relative to Business Cycle rule, 1-year horizon

(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Panel (a) shows the cumulative probability of the value of withdrawals under different rules relative to the Business Cycle rule. In Panel (b), dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

The chart shows that there is an essentially zero probability of ABC rule withdrawals undershooting the Business Cycle rule by 5% or more. The probability of withdrawals undershooting by 5% or more for the Preservation rule is around 60%, and above 75% for the Cash Flow rule. We saw earlier that withdrawals implied by expected returns or cash flows can be substantially below those under the Business Cycle rule. The chart shows that adjustments between -5% and -30% are very likely for the Preservation and Cash Flow rules. The ABC rule limits the reductions in withdrawals relative to the Business Cycle rule, where there is a minimal chance of a reduction in withdrawals below -5%.

The ABC rule accommodates cyclicality in withdrawals while also attempting to improve the distribution of fund values over the long term. Figure 10 Panel (b) shows the 20-year distributions of fund values. By targeting reductions in the deficit, the fund value distribution under the ABC rule shifts to the right of the Business Cycle rule. There remains a slightly higher probability of fund depletion relative to the Preservation rule, but the probability is lower than for the Business Cycle rule. The parameters of the ABC rule could be adjusted to further reduce the likelihood of depleting the fund, by more aggressively reducing the fiscal deficit.

[31] In addition, any real-world implementation would require more careful calibration and robustness analysis.

The ABC rule is closer to the Preservation rule in terms of its likelihood of fund depletion. The two rules achieve this in different ways, however. The ABC rule allows for cyclical spending at the cost of lower spending on average. For the Preservation rule, spending needs to be cut whenever the value of the fund falls. This implies that the ABC rule reallocates spending cuts from states of the world with low fund values to states of the world where deficit reduction is more feasible from a fiscal point of view. As a consequence of this, the ABC rule generates a wider distribution of fund values compared to the Preservation rule.[32]

While expected returns and the fund value are not explicitly included in the ABC rule, they will still matter for long-horizon outcomes. In particular, the degree to which the fund distribution under the ABC rule shifts rightward will depend on how the paths for expected returns and fiscal deficits compare. The more the target deficit undershoots withdrawals based on expected returns, the more the fund distribution under the ABC rule will lie to the right of the Business Cycle rule. The calibration of the ABC rule therefore still needs to incorporate expected return considerations and oil revenue flows, and this calibration would need to be revisited periodically.

6. Summary

We have outlined a simulation model of equity and fixed income returns, the Norwegian fiscal budget, the Norwegian Krone exchange rate and petroleum revenues. We use the model to simulate the evolution of the GPFG and the Norwegian fiscal budget, to understand the trade-off between two broad aims of the fiscal spending rule in Norway. While spending in line with expected returns aims to preserve the real value of the fund, withdrawals from the fund contribute to counter-cyclical budget deficits in Norway. Using the simulation framework, we illustrate how a misalignment can arise between the objectives of sustainable and cyclical withdrawals. When spending in line with expected returns, fund withdrawals do not match withdrawals required to finance fiscal deficits determined by the Norwegian business cycle. When withdrawals are used to accommodate counter-cyclical spending, there is a higher probability of depleting the real value of the fund over the long term.

We show that commonly proposed rules for guiding withdrawals, such as spending cash flows or smoothing spending, are also subject to this trade-off. We outline an alternative rule that aims to address short-term cyclical requirements, while attempting to preserve long-term values of the fund. This rule aims to reduce the fiscal deficit over time, and the target deficit and pace of reductions can be adjusted to balance risks across the short and long term. Withdrawals based on this rule are closely aligned with the Norwegian business cycle, and they do not directly depend on the value of the fund. Since the rule sets withdrawals in line with GDP, the average level of withdrawals needs to be reduced over time. This shifts the distribution of the fund to the right, reducing the likelihood of depleting the fund over the long term.

[32] Under the ABC rule, lower long-term spending effectively builds a 'buffer' that is used to cover cyclical spending needs. This makes depletions in fund values less likely.

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Appendix A: Simulation Model - US macro and asset prices

Below, we outline the simulation model for US macro variables and asset prices. For a more detailed discussion around the specification and calibration of this framework, we refer the reader to NBIM (2023).

The realised change in the log level of real output, denoted by z_t, is modelled with persistent and transitory components, τ_t^z and a_t, respectively:

$$\begin{aligned} z_t &= \tau_t^z + a_t \\ \tau_t^z &= (1-\rho_z)\,\mu_z + \rho_z \tau_{t-1}^z + \varepsilon_t^z \\ a_t &= \varepsilon_t^a, \end{aligned}$$

The realised change in the log price level, denoted as π_t, is also modelled with persistent and transitory components:

$$\begin{aligned} \pi_t &= \tau_t^\pi + c_t \\ \tau_t^\pi &= (1-\rho_\pi)\,\mu_\pi + \rho_\pi \tau_{t-1}^\pi + \varepsilon_t^\pi \\ c_t &= \rho_c c_{t-1} + \varepsilon_t^c. \end{aligned}$$

Long-term expectations of the persistent component of each macro variable using a constant-gain learning rule. Investors apply the rule as follows:

$$\begin{aligned} \bar{\tau}_t^z &= \nu_z \bar{\tau}_{t-1}^z + (1-\nu_z)\,z_t \\ \bar{\tau}_t^\pi &= \nu_\pi \bar{\tau}_{t-1}^\pi + (1-\nu_\pi)\,\pi_t. \end{aligned}$$

Investors also perceive transitory components of macro variables that differ from the underlying components, a_t and c_t. We define these real-time estimates of transitory components as:

$$\begin{aligned} \bar{a}_t &= z_t - \bar{\tau}_t^z \\ \bar{c}_t &= \pi_t - \bar{\tau}_t^\pi. \end{aligned}$$

For the calibration of long-term averages of output growth and inflation, μ_z and μ_π, we calibrate in line with weighted average values for G4 countries. The values are 1.5% and 2.0% for μ_z and μ_π, respectively.

We apply the following yield curve modelling approach to both the US and Norway. Below we describe the specification for the US. We construct yield curves for Norway using macro trends and short rate and term premium processes with the same specification as for the US, but calibrated to match cross-country co-movement in yields. The equilibrium real interest rate, r_t^*, combines output growth expectations with a convenience yield, and γ_t, the “real rate gap” (assumed zero for Norway).

$$r_t^* = \bar{\tau}_t^z - s_t - \gamma_t$$
$$s_t = (1-\rho_s)\mu_s + \rho_s s_{t-1} + \epsilon_t^s$$
$$\gamma_t = (1-\rho_\gamma)\mu_\gamma + \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma$$

A monetary policy rule describes the short-term nominal interest rate, i_t, is:

$$i_t = i_t^* + \phi_c \bar{c}_t + \phi_a \bar{a}_t + \phi_i \bar{i}_{t-1} + u_t^i,$$

where $i_t^* = \pi_t^* + r_t^*$ and $\bar{i}_t = i_t - i_t^*$. The term premium is represented through a one-factor structure:

$$x_t = (1-\rho_x)\,\mu_x + \rho_x x_{t-1} + \varepsilon_t^x.$$

The average level of term premium is determined by μ_x. With $\bar{X}_t = (\bar{i}_t, \bar{c}_t, \bar{a}_t, x_t)^{'}$, the n-period nominal zero-coupon yield, $y_t^{(n)}$, is given by:

$$y_t^{(n)} = i_t^* + a_n + b_n^{'} \bar{X}_t,$$

where a_n and b_n are recursions determined by the no-arbitrage condition imposed in the term structure model.

We model the value of the aggregate stock market index, P_t, and the corresponding index dividend, D_t. The index is the sum of the present values of all future dividends:

$$P_t = \sum_{n=1}^{\infty} P_t^{(n)}.$$

The price of the index dividend paid out n years from now, denoted by $P_t^{(n)}$, is the present value of D_{t+n}:

$$P_t^{(n)} = D_t \exp\left(n\left(g_t^{(n)} - y_t^{(n)} - \theta_t^{(n)}\right)\right),$$

where $g_t^{(n)}$ is the annualized expected dividend growth at the n-year horizon, $y_t^{(n)}$ is the n-year yield and $\theta_t^{(n)}$ is the risk premium compensating investors for dividend risk at the n-year maturity. Dividend growth is tied to nominal output growth:

$$g_t = \tau_t^z + \tau_t^\pi + (a_t + c_t)\,.$$

Risk premiums, θ_t, are comprised of persistent and transitory components, where θ_t^* and $\tilde{\theta}_t$ are the long-term and cyclical risk premiums, respectively:

$$\theta_t = \theta_t^* + \tilde{\theta}_t$$
$$\theta_t^* = \mu_{\theta^*} + \beta_{\theta^*} x_t$$
$$\tilde{\theta}_t = \rho_{\tilde{\theta}} \tilde{\theta}_{t-1} + \varepsilon_t^{\tilde{\theta}},$$

Appendix B: US calibration

Table 7 shows the historical and simulated moments for US macro variables and asset prices.

TABLE 7 Moments of macro variables and asset returns, historical vs. simulated (annualised)

Variable	σ (%, Hist.)	σ (%, Simulated)	AC (Hist.)	AC (Simulated)
		Panel A. Macro Variables		
Real GDP Growth	2.4	2.2	0.78	0.77
Inflation	2.3	2.2	0.97	0.93
		Panel B. Macro Trends		
Growth Trend ($\bar{\tau}_t^z$)	0.8	0.8	0.99	0.99
Inflation Trend ($\bar{\tau}_t^\pi$)	1.9	1.6	0.99	0.99
		Panel C. Bond Yields		
1Y Bond	3.5	2.3	0.96	0.97
10Y Bond	3.0	2.4	0.97	0.94
		Panel D. Asset Returns		
1Y Bond	2.4	1.4	0.39	0.68
10Y Bond	12.5	16.4	-0.05	0.08
30Y Bond	36.9	32.9	-0.05	0.00
Equities	15.8	15.7	-0.02	-0.06

NOTE: σ and AC refer to standard deviation and first-order autocorrelation, respectively. Observed moments are estimated using quarterly data over the period from Q1 1967 to Q3 2022. Sample period starts in Q1 1962 for one- and 10-year yield and in Q2 1977 for the remaining yields. Sample period ends in Q3 2022. Data are sourced from FRED.

Appendix C: Combining Expected Returns and Cyclical Spending

Below, we show the cyclical properties and long-term fund distributions for the following rule for withdrawals:

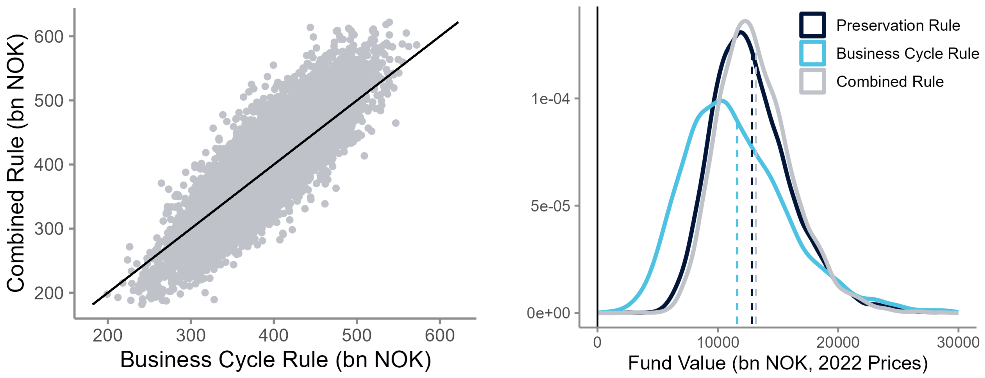
$$W_t^C = E_{t-1}\left(r_{t,\infty}^R\right) V_{t-1} + \left(\varepsilon_t^S - \varepsilon_t^R\right) Y_t.$$

This rule combines the Preservation rule with the cyclical component of the Business Cycle rule. Figure 11 shows the short-term properties of this combined rule in Panel (a) and the distribution of fund values at the 20-year horizon in Panel (b).

FIGURE 11 Short-term properties of fund withdrawals - Combined Rule

(A) Annual fund withdrawals: Combined Rule vs. Business Cycle rule

(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Panel (a) plots fund withdrawals over the first year of simulations for alternative rules. In Panel (b), dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

By incorporating expected returns into the combined rule, the distribution of fund values aligns with the Preservation rule over long horizons. The inclusion of cyclical spending increases the correlation with Business Cycle rule withdrawals. Variation in expected returns and fund values increases the variation around the 45-degree line, however. This illustrates the trade-off between cyclical and sustainable spending objectives.

Appendix D: Fund-smoothing rules

Here, we consider a rule that averages fund values over multiple periods when calculating fund withdrawals. Withdrawals, W_t^{AVG}, are set as follows:

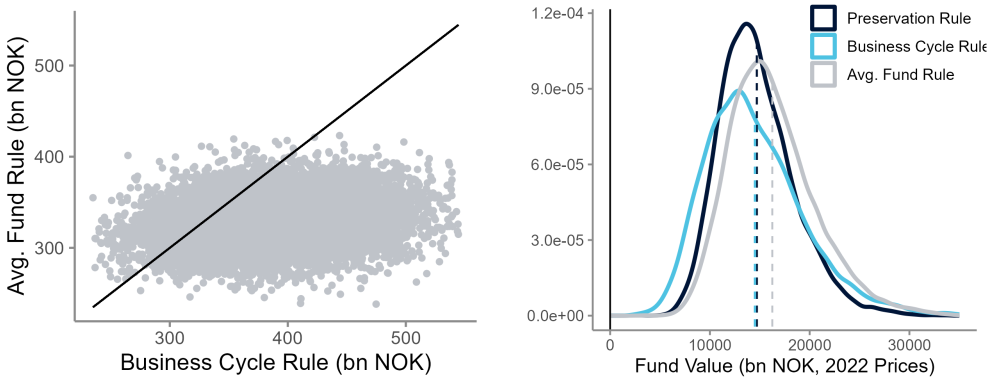
$$W_t^{AVG} = E_{t-1}\left(r_t^R\right)\frac{1}{A}\sum_{i=1}^{A}\left(V_{t-i-1}\right),$$

where A is the period of time over which the fund value is averaged, which we set equal to ten years. Figure 12 shows the short-term withdrawals for the smoothed fund rule compared to the Business Cycle rule.

FIGURE 12 Short-term properties of fund withdrawals - Average Fund Rule

(A) Annual fund withdrawals: Smoothed-fund vs. Business Cycle rule

(B) Fund value (bn NOK, 2022 prices) - 20-year horizon



NOTE: Panel (a) plots fund withdrawals over the first year of simulations for alternative rules. In Panel (b), dashed lines depict mean values. Real values are determined using the simulated price index for Norway.

In Panel (a), withdrawals based on the smoothed fund rule have a positive but low correlation with the business cycle rule withdrawals. Panel (b) shows the 20-year distributions of fund values, where the smoothed fund rule generates a distribution that sits a small amount to the right of the other distributions.